

VALUATION IN DERIVATIVES MARKETS

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ABN AMRO

Property Derivatives



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What is a Derivative?

- A contract that specifies the rights and obligations between two parties to receive or deliver future cash flows (or exchange of other securities or assets) based on some future event.
- Financial transaction whose value depends on the underlying value of the reference asset concerned.
- For example: the right (but not obligation) to buy 100 barrels of oil at \$80 per barrel in 2 years time is a call option.

Types of Derivatives

LINEAR

- Forwards
- Futures
- Swaps (most)

NON-LINEAR

- Vanilla Options, Warrants
- Exotic” Options (e.g. Barrier Options, Cliquet/Forward starts)
- Credit default “swaps” (are really options)

Uses of Derivatives

- Hedging
- Speculating
- Arbitrage
- Accessing remote markets
- Distribution of risk as securities i.e. “Securitisations”

Markets

- Interest Rates - Swaps, Futures, Options, FRAs, Exotics
- Foreign Exchange – Forwards, Swaps, Options, Exotics
- Equities (Shares) - Forwards, Swaps, Options, Exotics
- Credit – Vanilla Credit Default Swaps, CDOs (Tranched Default Swaps)
- Commodities- Forwards, Futures, Options
- Weather – Swaps
- Freight Routes – Forwards, Options
- Energy – Forwards, Options

- Emissions – Forwards
- Property – Swaps

MATURE MARKETS

NEW MARKETS

Yield Curve

- Discounting cash flows at different times is a routine but key part of derivatives valuations.
- Discount Rates for varying time periods can be implied from pure interest rate instruments such as Govt Bonds, Interest Rate Futures, Interest Rate Swaps, Treasury Bills, Money Market Deposits.
- Each could give slightly different 6 month interest rate for example.
- Market Convention is to use Money Market Deposits for early maturities (say 3 months), then Interest Rate Futures (say up to 18 mths) and then Interest Rate Swaps thereafter.
- Reasons: these are standard interest rate instruments used for hedging by the banks/market makers over these maturities.

Yield Curve

- Yield curves usually have upward sloping term structure; the longer the maturity, the higher the yield.
- For longer maturities, more catastrophic events might occur that may impact the investment, hence the need for a risk premium. Distant future more uncertain than the near future, and risk of future adverse events (e.g. default) being higher, hence liquidity premium.

New Derivatives Markets (e.g. Property Derivatives)

- Non standard deals/contracts
- Little or no “market making” with banks matching buy and sell trades exactly
- Low turnover
- Derivatives take pricing lead from physical market

Mature Markets (e.g. Interest Rates and Credit Derivatives markets)

- **Commoditisation/standardization**
- **Low transaction costs : narrow bid-offer spreads**
- **Very active trading by specialized traders**
- **Warehousing of risk/transactions by banks acting as “market makers”**
- **Derivatives trades exceed the trading in the underlying (“physical”)**
- **The derivatives market often leads the underlying (“physical market”) in terms of pricing.**

Valuation in Underdeveloped Market

Example: Property Index Swap.

Party A pays Party B the one year increase in the UK IPD Index

Party B pays Party A the market rate of interest + X %

- Hedging by Party A is inconvenient/expensive
- Therefore Party A “looks to” the expected performance/forecast of the UK property before deciding what X % should be.

Valuation in Mature Market (Linear instrument)

Example: Equity (Share) Swap.

Party A pays Party B the one year increase in Boots Plc shares

Party B pays Party A the market rate of interest + Y %

- Party A can hedge by buying the Boots Plc shares
- Pricing is based on the assumption that Party A will buy the share now and sell when the swap matures.
- Assume current price of Boots = S, dividends received over the year = D and interest rate = r% p.a.
- The cost of doing this trade is a function of the cost of buying and holding the share (net of dividends received). $Y = S * r - D$
- **KEY POINT** : Minimum price for the trade (ignoring party A's profit charge) is the cost of the RISK FREE REPLICATING PORTFOLIO.

Valuation in Mature Market (Non- Linear instrument)

Example: Equity (Share) Call Option.

Party A pays 15p (option premium) to Party B for the right to “call” (buy) Boots Plc shares at a predetermined price of 100p.

- Party A can hedge by buying the Boots Plc shares in advance but what happens if Party B does not exercise the option? Party A will be left with shares and associated price risk.
- So we need to introduce the concept of a DYNAMIC HEDGE in order to produce the risk free replicating portfolio.

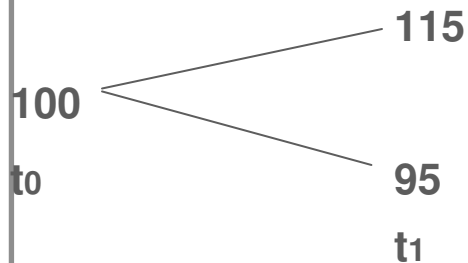
Simple Binomial Model used for Pricing Call Option

Current share price = 100 today

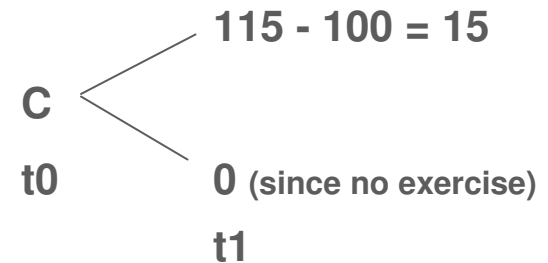
Suppose next day price will be 115 or 95 but we do not know probability

Pre-agreed exercise price of the call option is 100

Share



Call Option Values



Suppose we buy 0.75 share to hedge the call sold option

Portfolio valuation next day:

If share price rises = $0.75 (115) - 15 = 71.25$

If share price falls = $0.75 (95) - 0 = 71.25$

Simple Binomial Model used for Pricing Call Option

Since next day portfolio value is the same regardless of whether price rises or falls, we can say that we are hedged

Hedge ratio is 0.75 shares for every call option sold

We have succeeded in creating a **RISK FREE PORTFOLIO!**

So what? Well, if portfolio is risk free, then it must pay risk-free rate of interest (r)

Therefore the following relationship must hold:

$$\frac{\text{Next day Portfolio valuation}}{\text{Initial Investment}} = 1+r$$

$$\frac{71.25}{0.75 (100) - C} = 1+0.05$$

cost of shares premium received
 from call option sale

solving $C = 7.143$

that is minimum price of the call option is 7.143 p

Simple Binomial Model

Recap

To do this we needed:

- (i) risk-free rate of interest
- (ii) the possible share-price moves
- (iii) time period

Note: we did not know the probabilities of share price moves

- In order to apply binomial model to obtain realistic option prices, we need to divide time up into many small steps.
- As steps in time become shorter and shorter, discrete time will tend towards continuous time and the jumps or falls in share prices in each period will become infinitesimally small.
- For reasonably accurate results, the time to maturity should be divided into at least 50 steps.
- As we move along the binomial tree HEDGE RATIO CHANGES i.e. DYNAMIC hedge needs to be rebalanced.

The Black-Scholes Equation

- Black Scholes (1973) is of great importance in modern finance demonstrates the power of risk neutral (arbitrage) pricing.
- Why is B-S so great when we already have binomial model?
- Because it is an analytical solution (i.e. one-step) and therefore more elegant and computationally much more efficient.
- B-S makes one additional assumption that returns on assets are normally distributed.

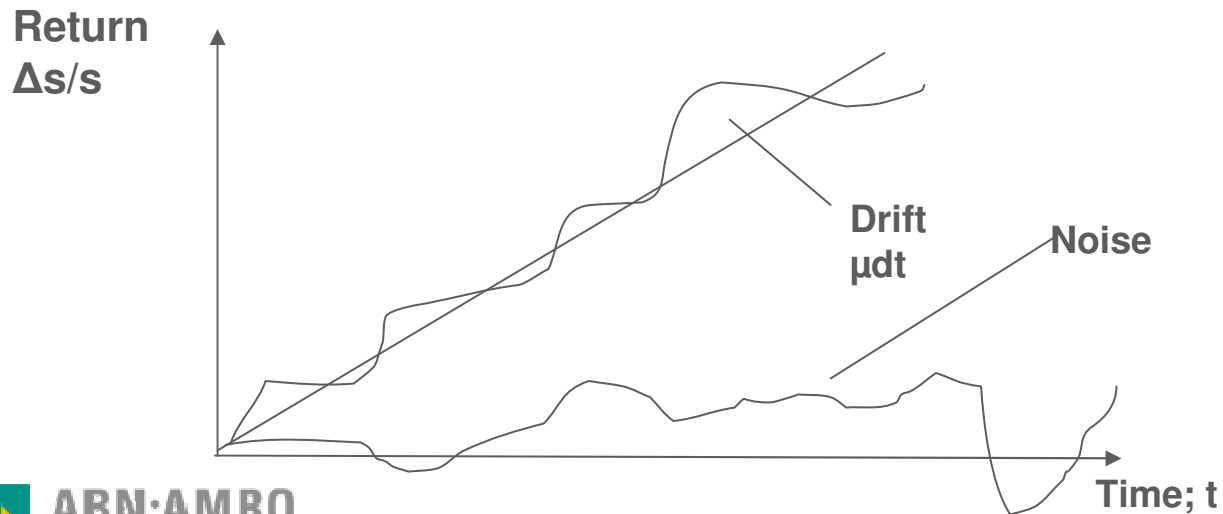
Stock Price Model

Stock price behaviour is assumed/modelled as geometric Brownian motion

$$\frac{\Delta s}{s} = \underbrace{\mu \Delta t}_{\text{constant return on stock or "drift"}} + \underbrace{\sigma \epsilon \sqrt{\Delta t}}_{\text{"noise" or Weiner process (Brownian motion)}}$$

$\frac{\Delta s}{s}$ is the % change in stock price.

ϵ is a random drawing from normal distribution



The Black-Scholes Equation

$\frac{\Delta s}{s}$ is normally distributed with mean $\mu \Delta t$ and standard deviation $\sigma \sqrt{\Delta t}$

$$\frac{\Delta s}{s} \sim \mathcal{N} \left(\underbrace{\mu \Delta t}_{\text{mean}}, \underbrace{\sigma \sqrt{\Delta t}}_{\text{std deviation}} \right)$$

By constructing risk neutral portfolio....

Black Scholes price of call option

$$c = S \cdot N(d_1) - ke^{-rT} N(d_2)$$

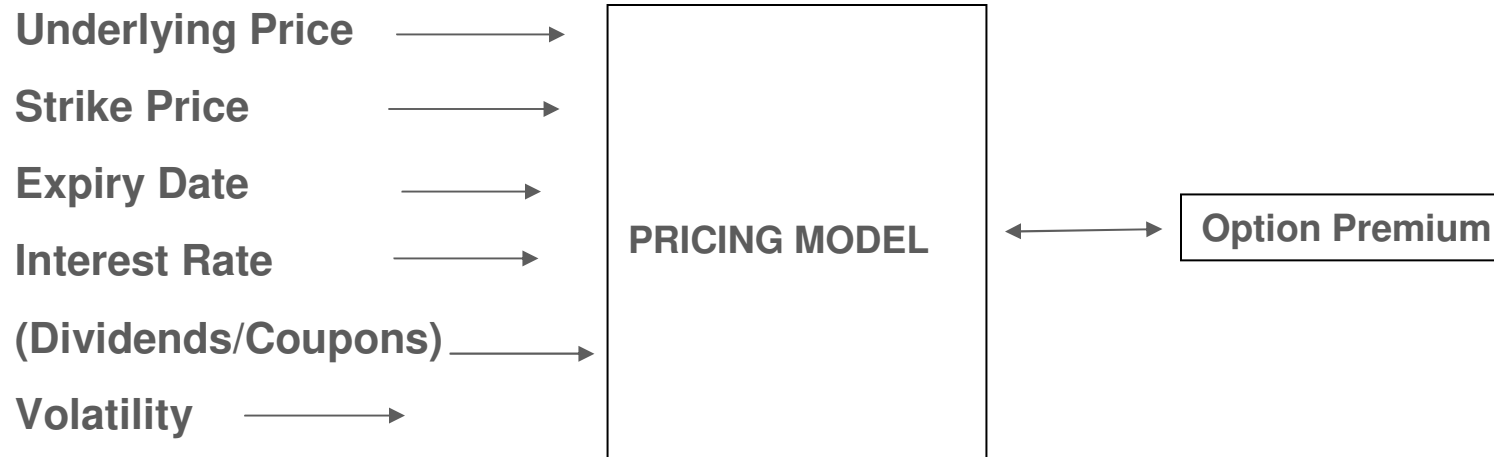
Black Scholes price of put option

$$p = ke^{-rT} N(d_2) - S N(-d_1)$$

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2) T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

Black Scholes- Vanilla Option Model



Black-Scholes Model assumptions

1. No transaction costs and market allows short selling
2. Trading and prices are continuous
3. Underlying asset prices follow geometric Brownian Motion
4. Interest rate during life of option is known and constant
5. The option can only be exercised on expiry

Dynamic Risk Management and “the Greeks”

- In practice, Risk Management is done on a portfolio basis with many different trades pooled together.
- The “Greeks” are sensitivities of option price to model inputs

First Order

- | | | |
|-------|---|--|
| Delta | - | measure of how option value changes with changes in underlying asset e.g. share price |
| Theta | - | measure of change in option value with change in time to maturity |
| Vega | - | change in option value with change volatility of underlying asset |
| Rho | - | change in option with respect to interest rates (in the case of a share option where delta relates to stock) |

Second Order

- | | | |
|-------|---|--|
| Gamma | - | Delta is <u>not</u> static. Gamma is a measure of how the delta itself changes with changes in |
|-------|---|--|

Derivatives valuation under IFRS

- IAS 39 requires that all derivatives are marked at fair value (mark to market)

IAS sets out a hierarchy for the determination of fair value:

- a) For instruments traded in active markets, use a quoted price.
- b) For instruments for which there is not an active market, use a recent market transaction.
- c) For instruments for which there is neither an active market nor a recent market transaction, use a valuation technique.

Implications:

- A large section of the derivatives market use valuation techniques with parameters implied from known points.
- If valuation technique cannot be calibrated back to market (e.g. complex products) then fair value is the price at which the trade was done -> Initial “Day One” P&L cannot be recognised.

Valuation in Derivatives Markets

- In an active, efficient market, valuation is based on the RISK FREE, REPLICATING PORTFOLIO
- Traders are employed to manage the exposure to the underlying exposure to the derivatives portfolio.
- For LINEAR Derivatives (Swaps, Futures) hedging is usually STATIC since future exchange is certain.
- For NON- LINEAR Derivatives (Options), exercise is not certain so hedging is dynamic.